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On the possibility of additional universal inverse-square forces arising from an asymmetric Maxwell theory

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Abstract. A 'Maxwell' theory that accommodates a possible attraction–repulsion-asymmetry (ARA) is seen to emerge from a suitably chosen variational principle. For 'neutral' bodies the theory is seen to reduce to a vector-gravitational type field with its coupling constant (G_{ARA}) related to the asymmetry parameter of the theory. The implications of the emerging analysis to some aspects in astronomy and to physics in general are discussed. It is seen that such an asymmetry could lead to:

(1) Changes in perihelian advance of gravitational orbits by an amount $\sim\frac{1}{3}G_{\mathrm{ARA}}/G$ times the general relativistic value.

(2) The associated waves carrying positive energy in opposition to the negative energy carried by waves in ordinary vector gravitational field theories.

(3) Interesting properties of the charges related to associated field theories.

As an *ex post facto* exposition of our analysis, we consider the following manifestly covariant form of an action:

$$I=-\frac{1}{16\pi}\sum_{(\alpha)(\beta)}(\alpha)g_{(\beta)}{}^{(\alpha)}\int F^{(\alpha)}_{\alpha\beta}F^{(\beta)\alpha\beta}\,\mathrm{d}^4x+\frac{1}{2}m\int\frac{\mathrm{d}z^{\mu}}{\mathrm{d}\tau}\frac{\mathrm{d}z_{\mu}}{\mathrm{d}\tau}\mathrm{d}\tau$$
$$+\sum_{(\alpha)}(\alpha)e^{(\alpha)}\int\frac{\mathrm{d}z^{\mu}}{\mathrm{d}\tau}A_{\mu}{}^{(\alpha)}(z),\qquad(1)$$

where the coordinates z^{μ} describe a test particle of mass m and charge $q^{(\alpha)}$. $F^{(\alpha)}_{\alpha\beta}$ is equal to $A^{(\alpha)}_{\beta,\alpha}-A^{(\alpha)}_{\alpha,\beta}$; (α), when occurring by itself, will take the values $+1$ or -1, and as a superscript it will by convention be used to label the two classes of charges as described below. The matrix $g_{(\beta)}{}^{(\alpha)}$ is defined as having components $\binom{1\;\;-k}{k\;\;-1}$, its inverse being $1/(1-k^2)$ of itself; k is a constant.

The associated Lagrangian describes (as shall soon be elaborated) two massless vector fields in interaction with charges that are separable into two classes—conventionally defined as positive- and negative-charged classes. The matrix $g_{(\alpha)}{}^{(\beta)}$ with $k\neq 1$ introduces the essential deviation from the Maxwell Lagrangian and has been constructed so that the mechanical effects of the field on a test particle of either class *cannot* be ascribed to the notion of a unique electromagnetic field independent (as in the Maxwell theory) of the class to which the test particle belongs.

To see this in detail, we obtain the field equations by varying the action with respect to $A^{(\alpha)\alpha}$ to give

$$\partial^{\beta}F^{(\alpha)}_{\beta\alpha}=4\pi\sum_{(\beta)}g_{(\beta)}{}^{(\alpha)}J_{\alpha}{}^{(\beta)},\qquad(2)$$

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where

$$J_\alpha^{(\beta)}(x) \equiv \int e^{(\beta)} \frac{\mathrm{d}z_\alpha}{\mathrm{d}\tau} \delta^4(x - z(\ )) \, \mathrm{d}\tau$$

represents the current-density four-vectors arising from the contribution from the (β) class of charges. The effect of the field on a test particle that may in general consist of contributions $(q+, q-)$ from both classes is obtained by varying $z^\mu(\tau)$ in the actions, giving

$$m\ddot{z}_\mu = \sum_{(\alpha)} (\alpha) e^{(\alpha)} z^\nu F_{\mu\nu}^{(\alpha)}. \tag{3}$$

The dots denote differentiation with respect to the parameter τ.

The action (1) (and therefore equations (2) and (3)) is seen to be invariant under $A^{(\alpha)\mu} \to A^{(\alpha)\mu} - \partial^\mu A^{(\alpha)}$—a freedom that may be used to define a 'Lorentz gauge' in our theory to yield

$$\Box A_\alpha^{(\alpha)} = 4\pi \sum_{(\beta)} g_{(\beta)}{}^{(\alpha)} J_\alpha^{(\beta)}. \tag{4}$$

An investigation of the static solutions of this equation coupled with equation (3) vivify the basic features of the theory, namely:

(1) The particles of each class (separately) interact with particles of their own class by laws legislated by the Maxwell theory;

(2) The crossed interaction of two particles belonging to different classes is represented by a force that is opposite in direction, but *not* equal in magnitude to the force between similar particles in the same class. This leads to the attraction–repulsion-asymmetry that is responsible for the title of this paper.

It seems to be quite natural to expect such an asymmetry in nature. Reflecting upon the (200 year old) notion of 'definition' of unit charges, we recall that they are defined as charges that repel similar charges by a unit force when a certain distance apart. To require further that unit charges of different classes so defined should attract each other by a force that may be equal in magnitude to that arising out of repulsion of unit charges in the same class is clearly out of the realm of a 'definition' of such charges, and is prone only to experimental investigation. We shall soon look into the difficulties one may encounter in a typical experiment required to test such an asymmetry. The assumption of an equality of attraction and repulsion is what leads to a simplification in the Maxwell theory rendering both classes of charges describable in a 'unified' way by defining one class to constitute 'negative' charges and the other as 'positive'. The cumulative effect of a concentrated assembly of such charges is then the same as that of a charge equal to that of their algebraic sum. This facility is not available in our (ARA) theory. The attraction/repulsion are put into the theory 'by hand' through the parameters (α), the charges of both classes are defined to be 'positive' and the matrix $g_{(\alpha)}{}^{(\beta)}$ with $k \neq 1$ expresses the inequality of the attraction and repulsion.

An experimental set-up to determine the deviation of k from unity would require a detailed knowledge of charges of both classes present in the source of the field required to examine the motion of charges. It should be noted that the use of a 'test charge' to define the source density would not do, as that involves the measurement of a Maxwell-free-charge density rather than the density of both classes. Moreover in a typical (Millikan experiment type) set-up, it is quite difficult to study a single isolated charge. Instead we obtain ionised 'drops' containing a large (uncertain) number of particles, the lack of knowledge of which undermines a direct evaluation of k.

Accepting the difficulties in the resolution of these problems, we shall now collect expressions in the (ARA) theory vis-à-vis classical electrodynamics and see how the motion of neutral bodies can be used to determine the upper limits to the deviation of k from unity. We first note that it is straightforward to define and interpret quantities $\boldsymbol{E}^{(\alpha)}$ and $\boldsymbol{B}^{(\alpha)}$ that reduce equation (2) to a form akin to the vintage form of the Maxwell theory. For example, the Biot–Savert law can be 'consistently derived' (see Jackson 1975) to be of the form $\mathrm{d}\boldsymbol{B}^{(\alpha)} = \sum_{(\beta)} (\mathrm{d}\boldsymbol{l} \otimes \boldsymbol{x}/c|\boldsymbol{x}|^3) g_{(\beta)}{}^{(\alpha)} I^{(\beta)}$, $I^{(\beta)}$ being the current of the (β) class of charges in the element $\mathrm{d}l$. From equation (2) follows the equation of continuity for each class of charges:

$$J^{(\beta)\alpha}{}_{,\alpha} = 0, \qquad \text{or} \qquad \boldsymbol{\nabla} \cdot \boldsymbol{J}^{(\alpha)} + \partial\rho^{(\alpha)}/\partial t = 0. \tag{5}$$

In the Maxwell theory the continuity equation was satisfied by $J^{[+]\alpha} - J^{(-)\alpha}$; here it is satisfied by both $J^{(+)\alpha}$ and $J^{(-)\alpha}$ separately. Clearly, as such, the ARA theory is inadequate to describe a 'charge conservation' in the presence of processes such as the weak decays of neutral 'elementary' particles and consequently the U(1) ARA theory needs to be extended to the gauge theories to include such processes.

Further, we can define an electric dipole–moment density $\boldsymbol{P}^{(\alpha)} \equiv \int \boldsymbol{x}'\rho^{(\alpha)}(x')\,\mathrm{d}^3x'$ and follow up by defining a displacement field by

$$\boldsymbol{D}^{(\alpha)} = \boldsymbol{E}^{(\alpha)} + \sum_{(\beta)} 4\pi g_{(\beta)}{}^{(\alpha)} \boldsymbol{P}^{(\beta)}, \qquad \text{to give } \boldsymbol{\nabla} \cdot \boldsymbol{D}^{(\alpha)} = 4\pi g_{(\beta)}{}^{(\alpha)} \rho^{(\beta)}$$

as the ARA Maxwell equation for macroscopic media. Similarly, we can define magnetic moment density (magnetisation) as $\frac{1}{2}(\boldsymbol{x} \otimes \boldsymbol{J}^{\alpha}(x))$ and $\boldsymbol{H}^{(\alpha)} \equiv \boldsymbol{B}^{(\alpha)} - \sum_{(\beta)} 4\pi g_{(\beta)}{}^{(\alpha)} \boldsymbol{M}^{(\beta)}$ to yield the following form for the field equations:

$$\begin{aligned} &\boldsymbol{\nabla} \cdot \boldsymbol{D}^{(\alpha)} = 4\pi \sum_{(\beta)} g_{(\beta)}{}^{(\alpha)} \rho^{\beta}, \qquad \boldsymbol{\nabla} \otimes \boldsymbol{H}^{(\alpha)} = \frac{4\pi}{c} \sum g_{(\beta)}{}^{(\alpha)} \boldsymbol{J}^{(\beta)} + \frac{1}{c}\frac{\partial \boldsymbol{D}^{(\alpha)}}{\partial t}, \\ &\boldsymbol{\nabla} \cdot \boldsymbol{B}^{(\alpha)} = 0, \qquad \boldsymbol{\nabla} \otimes \boldsymbol{E}^{(\alpha)} + (1/c)\partial \boldsymbol{B}^{(\alpha)}/\partial t = 0. \end{aligned} \tag{6}$$

Using these in exactly the same way as the expressions in the Maxwell theory gives the following forms for the energies associated with the 'electric' and the 'magnetic' field and an associated 'Poynting vector' for a typical ARA system:

$$w_{\mathrm{el}} = \frac{1}{8\pi(1-k^2)} \sum_{(\alpha)(\beta)} (\alpha) \boldsymbol{E}^{(\alpha)} \cdot \boldsymbol{D}^{(\beta)} g_{(\beta)}{}^{(\alpha)}, \tag{7}$$

$$w_{\mathrm{mag}} = \frac{1}{8\pi(1-k^2)} \sum_{(\alpha)(\beta)} (\alpha) g_{(\beta)}{}^{(\alpha)} \boldsymbol{H}^{(\beta)} \cdot \boldsymbol{B}^{(\alpha)}, \tag{8}$$

$$\boldsymbol{S}_{\mathrm{Poynting}} = \frac{c}{4\pi(1-k^2)} \sum_{(\alpha)(\beta)} (\alpha) g_{(\beta)}{}^{(\alpha)} \boldsymbol{E}^{(\alpha)} \otimes \boldsymbol{H}^{(\beta)}. \tag{9}$$

The stress–energy tensor T_{ij}, consistent with an expected form of momentum conservation law

$$\frac{\mathrm{d}}{\mathrm{d}t}(P_{\mathrm{mechanical}} + P_{\mathrm{field}})_i = \sum_j \int \frac{\partial}{\partial x_j} T_{ij}\,\mathrm{d}^3x,$$

can be seen to be of the form

$$T_{ij}=\frac{1}{4\pi(1-k^2)}\sum_{(\alpha)(\beta)}(\alpha)g_{(\beta)}{}^{(\alpha)}[E_i^{(\alpha)}E_j^{(\beta)}+B_i^{(\alpha)}B_j^{(\beta)}$$
$$-\tfrac{1}{2}(\boldsymbol{E}^{(\alpha)}\cdot\boldsymbol{E}^{(\beta)}+\boldsymbol{B}^{(\alpha)}\cdot\boldsymbol{B}^{(\beta)})\delta_{ij}]. \tag{10}$$

The complete stress tensor is given by

$$T_\mu^\nu=\sum_{(\alpha)(\beta)}(\alpha)\frac{g_{(\beta)}^{(\alpha)}}{(1-k^2)4\pi}(-F^{(\alpha)\nu\alpha}F_{\mu\alpha}^{(\beta)}+\tfrac{1}{4}g_\mu^\nu F^{(\alpha)\alpha\beta}F_{\alpha\beta}^{(\beta)}). \tag{11}$$

The analysis finds an interesting application to *electrodynamics of neutral bodies*: we consider an idealised body which is *uniformly* (as opposed to overall) neutral, i.e. contains equal amounts of positive- and negative-charged-class particles in each of its parts. This is represented by $J_\mu^{(\alpha)}=J_\mu^{(-\alpha)}$ for the source-current density, which in turn implies $A_\mu^{(\alpha)}=A_\mu^{(-\alpha)}\equiv(1-k)A_\mu$ in the (Lorentz) gauge responsible for equation (4). If in addition we consider the source to be a dust distribution, then the total amount of charge of each class in every volume element of it is proportional to the total mass. In particular, we can see that if there are two basic (equally charged) particles (one in each class) then their charges are $e^{(+)}=e^{(-)}=cm$, where m is the total mass of a neutral dust configuration $(e^{(+)}, e^{(-)})$ and equals the sum of the rest masses of each of the particles, and c is a constant determined by the charge to mass ratios of the two particles. Bearing all this in mind, the action in equation (1) reduces to

$$I=\frac{1}{16\pi G_{\mathrm{ARA}}}\int G_{\mu\nu}G^{\mu\nu}\,\mathrm{d}^4x+\frac{m}{2}\int\frac{\mathrm{d}z^\mu}{\mathrm{d}\tau}\frac{\mathrm{d}z_\mu}{\mathrm{d}\tau}\mathrm{d}\tau+m\int\Phi_\mu\frac{\mathrm{d}z^\mu}{\mathrm{d}\tau}\mathrm{d}\tau, \tag{12}$$

where we have defined $2c(1-k)A_\mu\equiv\Phi_\mu$, $G_{\mu\nu}\equiv\Phi_{\nu,\mu}-\Phi_{\mu,\nu}$ and $G_{\mathrm{ARA}}\equiv 2c^2(1-k)$. This represents an action of a vector gravitational field (Misner *et al* 1973), the essential feature being that the associated vector 'gravitational waves' carry a positive energy in our case (being given by equations (7) and (8)), unlike the case discussed by Misner *et al* (1973). We shall now see how the presence of such asymmetry introduces changes in 'planetary orbits' over and above the changes brought about by the general relativity theory.

To this effect we can easily see, by following standard methods (quite like those used to derive the metric for a Reisner–Nordström black hole (see e.g. Jeffrey 1921)), that equating the stress tensor (equation (11)), for the case we are considering, to $1/8\pi G\times$ the Einstein tensor gives the following metric solution for the most general spherically symmetric case:

$$\mathrm{d}s^2=\gamma\,\mathrm{d}t^2-\gamma^{-1}\,\mathrm{d}r^2-r^2\,\mathrm{d}\theta^2-r^2\sin^2\theta\,\mathrm{d}\phi^2 \tag{13}$$

with $\gamma=1-2Gm/r+2e^2(1-k)^2/(1+k)r^2$; using $e=cm$, the second term in γ is seen to be proportional to $(G_{\mathrm{ARA}}m/r)^2$. Neglecting this term and using the equation of motion

$$\frac{\mathrm{d}^2x^\alpha}{\mathrm{d}s^2}+\Gamma^\alpha_{\mu\nu}\frac{\mathrm{d}x^\mu}{\mathrm{d}s}\frac{\mathrm{d}x^\nu}{\mathrm{d}s}=\sum_{(\alpha)}(\alpha)e^{(\alpha)}F^{(\alpha)\alpha\beta}\frac{\mathrm{d}x_\beta}{\mathrm{d}s}, \tag{14}$$

we can see after some tedious but straightforward algebra (see Rindler 1977) that a typical orbit is represented by

$$\mathrm{d}^2u/\mathrm{d}\varphi^2+u=m(G+G_{\mathrm{ARA}})/h^2+3Gmu^2+G_{\mathrm{ARA}}(G_{\mathrm{ARA}}-2G)M^2u/h^2, \tag{15}$$

where $u \equiv 1/r$ and h is an integration constant with the dimensions of angular momentum. Identifying $G + G_{\text{ARA}}$ as the observed gravitational constant, we can see that the last term introduces an additional change in the perihelion advance. The good agreement of the perihelion observation experiments (about 2% accuracy according to the precision of the early 1970's (Misner *et al* 1973, p 1048)) forces us to limit G_{ARA} so it does not imply a perihelion advance of say more than the order of 1%. Therefore, using $G_{\text{ARA}} \ll G$ we find the solution of equation (15) to be given by

$$u(\varphi) = \frac{u_0(\phi')}{(1 - 2G_{\text{ARA}}GM^2/h^2)} \equiv \frac{u_0(\phi')}{Y}, \tag{16}$$

where $\phi' \equiv \sqrt{y}\phi$ and $u_0(\phi)$ is the solution with $G_{\text{ARA}} = 0$, i.e. the Einstein orbit. For $y \approx 1$ this gives a change of perihelion by an amount $G_{\text{ARA}}GM^2/h^2$. Recalling the Einstein result $3m^2G^2/h^2$ we see that the result is $\frac{1}{3}G_{\text{ARA}}/G \times$ Einstein's result. An accuracy of 1% in observation indicates an upper limit $G_{\text{ARA}} < 0{\cdot}03G$ which in turn asks for a very small asymmetry $|1-k| \lesssim 10^{-38}$ (where we have used $G_{\text{ARA}} = 2C^2(1-k)$ and C to approximate the charge to mass ratio of the proton). The asymmetry could however play a measurable role in cosmology.

Finally, a field endowed with charges with asymmetric interaction can have interesting consequences. Consider a complex scalar field, for example, in which the invariance of the Lagrangian $-m^2\phi^*\phi - \partial_\mu\phi^*\partial_\mu\phi$ under the transformation $\phi \to \phi\ e^{ic}$ urges the existence of a conserved quantity $s_\mu = i(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi)$. Although there is nothing compelling in the formulation so far (Sakurai 1973), we relate s_μ to an 'electromagnetic current' by $j_\mu^{(\alpha)} = e^{(\alpha)}s_\mu$. Considering an interaction with an external ARA potential $A_\mu^{(\alpha)}$, a minimal replacement $\partial_\mu \to \partial_\mu - i(\alpha)e^{(\alpha)}A_\mu^{(\alpha)}$ in the Lagrangian gives the interaction between the external potential and the (α) class of charge by the interaction Lagrangian:

$$L_{\text{int}} = -(\alpha)ie^{(\alpha)}\phi^*A_\mu^{(\alpha)}\partial_\mu\phi + (\alpha)ie^{(\alpha)}\partial_\mu\phi^*A_\mu^{(\alpha)}\phi - e^{(\alpha)2}\phi^*A_\mu^{(\alpha)}A_\mu^{(\alpha)}\phi.$$

In the ARA theory $A_\mu^{(-\alpha)}$ is in general not equal to $A_\mu^{(\alpha)}$. Therefore the result that 'if ϕ is a solution with an external potential A_μ $(=(0, 0, 0iA)$ say) then ϕ^* is a solution with A_0 replaced by $-A_0$' does not hold. Thus if ϕ is a field corresponding to a charge $e^{(\alpha)}$ then ϕ^* is not a field corresponding to a charge in the other class; it is just interpretable as a time-reversed state of the charge of the same class.

We conclude that the number operators $N^{(+)}$ and $N^{(-)}$ occurring in a typical expression for the charge operator $Q^{(\alpha)} = e^{(\alpha)}\sum_k N_k^+ - N_k^-$ both refer to the same class (α) of charges. In the Dirac theory of the electron the 'holes' in the redefined vacuum are not interpretable as charges of the 'positive' ARA class, but just the time reversals of the negative ARA class. It could well be that most of observed nature consists of charges of one class (electrons) and heavier time-reversed entities (protons) of charges of the same class. In that case, however, we cannot preclude the existence of the other charged state and its time-reversed entities. The limitation of k by the astronomical observations may then be erroneous and k could then be significantly different from unity.

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